



(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199208

Roll No.

| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|
| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|

B. Tech.

(SEM. II) THEORY EXAMINATION, 2014-15
MATHEMATICS - II

Time : 3 Hours]

[Total Marks : 100

Note : Attempt all questions.

SECTION A

1 Attempt all parts of this question : **10×2=20**

(a) Solve : $(2D-1)^2 y = 0$ where $D = \frac{d}{dt}$.

(b) Find the P.I. of $D^2 y = x^2 + 2x + 1$.

(c) Write the generating function for $P_n(x)$.

(d) Determine the expressin for $J_{1/2}(x)$.

(e) Find the Laplace transform of $t^2 e^{-3t}$.

- (f) Find the function whose Laplace transform is $e^{-\pi s} / s^2$.
- (g) Find the constant term if $f(x) = x^2$ is expanded in Fourier series in $(-\pi, \pi)$.
- (h) Solve $(D^2 - 2DD' + D'^2)z = 0$
- (i) Classify $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$
- (j) Write one dimensional wave equation and its solution.

SECTION B

2 Attempt any three parts of the following : **3×10=30**

- (a) Solve the following system of simultaneous differential equations

$$\frac{dx}{dt} + x - 2y = 0, \quad \frac{dy}{dt} + x + 4y = 0;$$

$$x(0) = y(0) = 1.$$

- (b) Solve in series : $2x^2 y'' + xy' - (x+1)y = 0$
- (c) Using Laplace transform solve :

$$\frac{d^2 y}{dx^2} + 9y = 6 \cos 3t; \quad y(0) = 2, y'(0) = 0$$

(d) Obtain the Fourier series for the function

$$f(x) = \begin{cases} x & , -\pi < x < 0 \\ -x & , 0 < x < \pi \end{cases}$$

hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

(e) Find the temperature in a bar of length 2 whose ends are kept at zero temperature and the initial

$$\text{temperature is } \sin \frac{\pi x}{2} + 3 \sin \frac{5\pi x}{2}.$$

SECTION C

Note : Attempt any two parts from each question $(2 \times 5) \times 5 = 50$ of this section.

3 (a) Solve : $(D^2 + 5D - 6)y = \sin 3x$

(b) Solve by method of variation of parameters

$$\frac{d^2 y}{dx^2} + y = \sec x$$

(c) Solve : $\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = e^x \sec x$

4 (a) Prove that $P_n(-x) = (-1)^n P_n(x)$

(b) Prove that $\frac{d}{dx}(x^n J_n) = x^n J_{n-1}$

(c) Solve in terms of Bessel functions $4y'' + 9xy = 0$.

- 5 (a) Find the Laplace transform of $te^{-2t} \sin 3t$.
 (b) State and prove second shifting property.
 (c) Find the inverse Laplace transform of

$$\frac{15}{s^2 + 4s + 13}$$

- 6 (a) Find the half range Fourier sine series of

$$f(x) = x^2 \text{ in } 0 < x < 3.$$

(b) Solve : $x^2 p + y^2 q = (x + y)z$

(c) Solve : $(D^2 + 3DD' + 2D'^2)z = e^{2x-3y}$

- 7 (a) Solve by the method of separation of variables

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$$

- (b) Find the temperature distribution in a bar of length ℓ whose ends are kept at zero temperature and initial temperature is μ_0 .

- (c) Find the temperature distribution in a rod of length π which is totally insulated including the ends and the initial temperature distribution is $100 \cos x$.