B. Tech.

(SEM. IV) EXAMINATION, 2006 - 2007
COMPUTER BASED NUMERICAL METHODS
(COMMON TO CIVIL ENGG., CHEM. ENGG. & CHEM. TECH.)

Time : 3 Hours] [Total Marks : 100

Note : Answer all questions. Each question carry equal marks.

1. Attempt any four parts of the following : 5x4=20

(a) If $\pi = \frac{22}{7}$ is approximated as 3.14, find the absolute error, relative error and percentage error.

(b) Draw a flow chart to find real roots of the equation $ax^2 + bx + c = 0$.

(c) Perform five iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

(d) Explain the difference between regula-falsi method and secant method.

(e) Find the value of $(17)^{1/3}$ by Newton's Raphson method correct to 4 decimal places.

(f) Find a real root of the equation $x^3 + x^2 - 1 = 0$, by using iteration method.

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2 Attempt any two parts of the following : \[10 \times 2 = 20\]

(a) Solve the following system of equations by Gauss-Seidal method correct to three decimal places:

\[
\begin{align*}
  x + y + 54z &= 110 \\
  27x + 6y - z &= 85 \\
  6x + 15y + 2z &= 72
\end{align*}
\]

(b) For the systems of equations

\[
\begin{bmatrix}
  4 & -1 & 0 \\
  -1 & 4 & -1 \\
  0 & -1 & 4
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix}
= \begin{bmatrix}
  3 \\
  2 \\
  3
\end{bmatrix}
\]

(1) Find the optimal relaxation parameter \( w_{opt} \) for the SOR iteration scheme.

(2) Write the SOR iteration scheme in residual vector form, starting with \( x^{(0)} = \begin{bmatrix} 0.5, & 0.5, & 0.5 \end{bmatrix}^T \), iterate three times.

(c) Solve the system of equations:

\[
\begin{bmatrix}
  2 & 1 & -4 & 1 \\
  -4 & 3 & 5 & -2 \\
  1 & -1 & 1 & -1 \\
  1 & 3 & -3 & 2
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4
\end{bmatrix}
= \begin{bmatrix}
  4 \\
  -10 \\
  2 \\
  -1
\end{bmatrix}
\]

by the LU decomposition method assuming \( u_{ii} = 1 \), \( i = 1(1)4 \).

3 Attempt any two parts of the following : \[10 \times 2 = 20\]

(a) Express the value of \( \theta \) in terms of \( x \) using the following data:

<table>
<thead>
<tr>
<th>( x )</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>184</td>
<td>204</td>
<td>226</td>
<td>250</td>
<td>276</td>
<td>304</td>
</tr>
</tbody>
</table>

Also find \( \theta \) at \( x = 43 \) and \( x = 84 \).
(b) The population of a certain town is given below. Find the rate of growth of the population in 1941 and 1961:

<table>
<thead>
<tr>
<th>Year</th>
<th>Population in lacs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1931</td>
<td>40.62</td>
</tr>
<tr>
<td>1941</td>
<td>60.80</td>
</tr>
<tr>
<td>1951</td>
<td>17.95</td>
</tr>
<tr>
<td>1961</td>
<td>103.56</td>
</tr>
<tr>
<td>1971</td>
<td>132.65</td>
</tr>
</tbody>
</table>

(c) (1) The velocity $v$ of a particle at distance $S$ from a point on its path is given by the table below:

<table>
<thead>
<tr>
<th>$S$ in meter</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$ m/sec</td>
<td>47</td>
<td>58</td>
<td>64</td>
<td>65</td>
<td>61</td>
<td>52</td>
<td>38</td>
</tr>
</tbody>
</table>

Estimate the time taken to travel 60 meters by using Simpson's $\frac{1}{3}$ rule.

(2) Evaluate $\int_{0}^{2} \frac{dx}{x^2 + x + 1}$ to three decimals, dividing the range of integration into 8 equal parts.

4 Attempt any two parts of the following: $10 \times 2 = 20$

(a) (1) Solve $\frac{dy}{dx} = x + y$, given $y(1) = 0$ get $y(1.1)$ by Taylor's series method.

(2) Given $y' = -y$ and $y(0) = 1$, determine the value of $y$ at $x = (0.01)$ $(0.01)$ $(0.04)$ by Euler method.

(b) Using Runge-Kutta method of fourth order, find $y(0.8)$ correct to 4 decimal places if $y' = y - x^2$, $y(0.6) = 1.7379$, taking $h = 0.1$.

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(c) Using Milne’s method, find $y(2)$ if $y(x)$ is the solution of
\[
\frac{dy}{dx} = \frac{1}{2} (x + y) \quad \text{given}
\]
$y(0) = 2, \ y(0.5) = 2.636, \ y(1) = 3.595$ and $y(1.5) = 4.968$.

5 Attempt any two parts of following : $10 \times 2 = 20$

(a) Using power method, find numerically the longest eigen value of
\[
A = \begin{bmatrix}
25 & 1 & 2 \\
1 & 3 & 0 \\
2 & 0 & -4
\end{bmatrix}
\]
corresponding eigen vector.

(b) Solve the boundary value problem $u'' = u + x,$
$u(0) = 0, \ u(1) = 0$ with $h = \frac{1}{4}$, using second order method.

(c) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ for the square mesh given below with boundary values as shown, using Liebnann’s iteration procedure (perform five iterations only)

\[
\begin{array}{cccccc}
0 & 100 & 200 & 100 & 0 \\
200 & u_1 & u_2 & u_3 & 200 \\
400 & u_4 & u_5 & u_6 & 400 \\
200 & u_7 & u_8 & u_9 & 200 \\
0 & 100 & 200 & 100 & 0
\end{array}
\]

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