B. Tech.
(SEM. IV) EXAMINATION, 2006 - 2007
SIGNALS & SYSTEMS

Time : 3 Hours] [Total Marks : 100

Note : Attempt all questions.

1 Attempt any four parts of the following : 5x4=20
   (a) A discrete time signal \( x(n) \) is defined as
      \[
      x(k) = \begin{cases} 
      1 + \frac{k}{3}, & -3 \leq k \leq -1 \\
      1, & 0 \leq k \leq 3 \\
      0, & \text{otherwise}
      \end{cases}
      \]

      (1) Determine its values and sketch the signal \( x(k) \).
      (2) Sketch the signal \( x(-n+4) \)

   (b) For the following systems, determine whether or not the system is
      (1) Stable
      (2) Causal
      (3) Linear
      (4) Memory less
      (i) \( T[x(n)] = X(n-n_0) \)
      (ii) \( T[x(n)] = 3e^{x(n)} \)

V-3037] 1 [Contd...
(c) (1) Show that the $x(t) = e^{i\omega_0 t}$ complex exponential signal is periodic.

(2) Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods $T_1$ and $T_2$. Under what condition $S$ is the sum $x(t) = x_1(t) + x_2(t)$ periodic.

(d) Explain the properties of continuous time LTI system.

(e) Let $x(t) * h_1(t) = f_1(t)$ and $h_1(t) * h_2(t) = f_2(t)$ with LTI system show that $x(t) * f_2(t) = x(t) * \{h_1(t) * h_2(t)\}$

(f) Consider a sequence $x(n)$

$x(n) = 4 - n \quad 0 \leq n \leq 4$

$= 0$ otherwise

Find its discrete time Fourier transform $X(e^{j\omega})$.

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**2** Attempt any **four** parts of the following: $5 \times 4 = 20$

(a) Find the Fourier transform of

$x(t) = e^{-at} \quad \forall \ t \geq 0$

$= 0 \quad \forall \ t < 0$

(b) Describe the time domain properties of ideal frequency selective filters.

(c) Design a band pass filter that has the centre of its pass band at $\omega = \frac{\pi}{2}$. Zero in its frequency response characteristic at $\omega = 0$ and $\omega = \pi$ and its magnitude response is $\frac{1}{\sqrt{2}}$ at $\omega = \frac{4\pi}{9}$.

V−3037] [Contd...
(d) Determine the Fourier transform of the signal
\[ x(n) = \begin{cases} A, & -M \leq n \leq M \\ 0, & \text{elsewhere} \end{cases} \]

(e) Determine the output \( Y(n) \) of a relaxed linear time-invariant system with impulse response
\[ h(n) = a^n u(n), \quad |a| < 1 \] when the input is a unit step sequence, that is \( x(n) = u(n) \).

(f) Determine the Fourier transform of the function
\[ y(n) = x(n) \ast h(n). \]

3 Attempt any two parts of the following: \( 10 \times 2 = 20 \)

(a) (i) Show that distribution function
\[ F_X(x) = \int_{-\infty}^{x} f_X(x) \, dx \]
where \( f_X(x) \to -\infty \)

the density function of random variable \( x \).

(ii) A probability density function is given as
\[ f_X(x) = a e^{-b|x|} \]
\( X \) is the random variable, \( x = -\infty \) to \( x = \infty \). Determine the relationship between \( a \) and \( b \).

(b) A joint density function of the random variables \( X \) and \( Y \) is given as
\[ f_{XY}(x, y) = \begin{cases} e^{-(x+y)} & \text{for} \quad x \geq 0, \quad y \geq 0 \\ 0 & \text{otherwise} \end{cases} \]

Determine the followings:
(1) \( P(X < 1) \)
(2) \( P(X > Y) \)

(c) State different properties of probability density function and probability distribution functions.
4 Attempt any two parts of the following: \(10 \times 2 = 20\)
(a) State and prove sampling theorem.
(b) Compute the Fourier transform of the following signals:

1. \(x(n) = 2^n u(-n)\)

2. \(x(n) = \left(\frac{1}{4}\right)^n u(n + 4)\).

(c) Explain the discrete time processing of continuous time signal? To achieve this give the Block diagram of a system.

5 Attempt any two parts of the following: \(10 \times 2 = 20\)
(a) Find z-transform and also the frequency response of

\[ h(n) = \left(\frac{1}{2}\right)^n \left[\left(\frac{1}{2}\right)^n + \left(-\frac{1}{4}\right)^n\right] u(n) \]

locate the zeros and poles in \(z\)-plane.

(b) Determine the \(z\)-transform of the signals and ROC of the following:

1. \(x(n) = na^n u(n)\)

2. \(x(n) = (-1)^{n+1} \frac{a^n}{n} u(n-1)\)

(c) Using \(z\)-transform find the convolution two signals

\[ x_1(n) = \{1, -2, 1\} \]

\[ x_2(n) = \begin{cases} 1, & 0 \leq n \leq 5 \\ 0, & \text{elsewhere} \end{cases} \]