B. Tech.

(SEM. VI) EXAMINATION, 2006-07

PRINCIPLES OF OPERATIONS RESEARCH

Time : 3 Hours] [Total Marks : 100

Note : (1) Attempt all questions.
(2) All questions carry equal marks.
(3) The choice of questions is internal as indicated in each question.
(4) Graph papers will be provided on demand.

1 Attempt any four of the following : 5x4=20

(a) Solve the following LPP by graphical method :
Maximize \( Z = 3x_1 + 4x_2 \)
Subject to \( 4x_1 + 2x_2 \leq 80, \ 2x_1 + 5x_2 \leq 180 \)
and \( x_1, x_2 \geq 0 \)

(b) Explain clearly the following terms used in LPP :
(1) Objective function
(2) Decision variables,
(3) Slack variables
(4) Surplus variables and
(5) Redundant constraints.
(c) Convert the following LPP to the standard form:

Maximize \( Z = 5x_1 + 7x_2 + 9x_3 \)

Subject to
\[
\begin{align*}
4x_1 - 6x_2 &\leq 5 \\
3x_1 + 2x_2 + 7x_3 &\geq 12 \\
4x_1 + 3x_3 &\leq 2
\end{align*}
\]
and \( x_1, x_2 \geq 0 \)

(d) Write the dual of the following LPP:

Minimize \( Z = 3x_1 - 2x_2 + 6x_3 \)

Subject to
\[
\begin{align*}
4x_1 + 5x_2 + 3x_3 &\geq 7 \\
3x_1 + x_2 + 6x_3 &\geq 5 \\
7x_1 - 2x_2 - 3x_3 &\leq 10 \\
x_1 - 2x_2 + 5x_3 &\geq 3 \\
4x_1 + 7x_2 - 9x_3 &\geq 2
\end{align*}
\]
and \( x_1, x_2, x_3 \geq 0 \)

(e) Solve the following LPP by simplex method:

Minimize : \( Z = x_1 - 3x_2 + 3x_3 \)

Subject to
\[
\begin{align*}
3x_1 - x_2 + 2x_3 &\leq 7 \\
2x_1 + 4x_2 &\geq -12 \\
-4x_1 + 3x_2 + 8x_3 &\leq 10
\end{align*}
\]
and \( x_1, x_2, x_3 \geq 0 \)
(f) Fill in the blanks so that the following statements are correct:

1. A constraint which does not affect the solution of an LPP, is called ________

2. Any solution to an LPP which satisfies the non-negativity condition is called ________

3. The feasible solution of an LPP is ________ of the objective function.

4. Iso-profit lines on a graph of an LPP would always be ________ to each other.

5. A constraint $5x_1 + 3x_2 \leq 12$ on an LPP is replaced by the constraint $5x_1 + 3x_2 \leq 7$. This would make the LPP more restrictive in nature. The constraint $5x_1 + 3x_2 \leq 12$ now becomes ________.

2. Attempt any four of the following:

(b) What is a transportation problem? Give the mathematical formulation of the transportation problem.

(b) What is a trans-shipment problem? Explain how it can be formulated and solved as a transportation problem?
(c) Find the initial basic feasible solution for the following transportation problem by VAM:

<table>
<thead>
<tr>
<th>Factory</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>11</td>
<td>13</td>
<td>17</td>
<td>14</td>
<td>250</td>
</tr>
<tr>
<td>$F_2$</td>
<td>16</td>
<td>18</td>
<td>14</td>
<td>10</td>
<td>300</td>
</tr>
<tr>
<td>$F_3$</td>
<td>21</td>
<td>24</td>
<td>13</td>
<td>10</td>
<td>400</td>
</tr>
</tbody>
</table>

| Demand  | 200   | 225   | 275   | 250   | 950    |

(d) A company has five jobs to be done on five machines. Any job can be done on any machine. The cost of doing the jobs in different machines are given below:

<table>
<thead>
<tr>
<th>Jobs</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$M_4$</th>
<th>$M_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_1$</td>
<td>13</td>
<td>8</td>
<td>16</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>$J_2$</td>
<td>9</td>
<td>15</td>
<td>24</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>$J_3$</td>
<td>12</td>
<td>9</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$J_4$</td>
<td>6</td>
<td>12</td>
<td>10</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td>$J_5$</td>
<td>15</td>
<td>17</td>
<td>18</td>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

Assign the jobs for different machines so as to minimize the total cost?

V-9974] 4 [Contd...
(e) Solve the following integer programming problem using branch and bound method:

Maximize \( Z = 3x_1 + 2.5x_2 \)

Subject to \( x_1 + 2x_2 \geq 20, \quad 3x_1 + 2x_2 \geq 50 \)

and \( x_1, x_2 \geq 0 \) and both are integers.

(f) Give the various steps involved in Hungarian method to solve an assignment problem.

3 Attempt any **two** of the following: \(10 \times 2 = 20\)

(a) Calculate the variance and expected activity time for the activities of the network shown in the following figure:

For each activity, the three estimates \( t_0, t_m \) and \( t_p \) are given along the arrows in the \( t_0 - t_m - t_p \) order. Enter calculations in the tabular form.

(b) Consider **PERT** network given in the following figure:

V-9974] 5 [Contd...
Find the float of each activity and identify the critical path if the scheduled completion time for the project is 20 weeks. Also, identify the sub-critical path.

(c) There are five jobs, each of which has to go through the two machines $M_1$ and $M_2$ in the order $M_1M_2$.

Processing times are given in the following table:

<table>
<thead>
<tr>
<th>Processing times in hours for the job</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machines</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Machine - $M_1$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>Machine - $M_2$</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

Determine a sequence of the five jobs that will minimize the total elapsed time $T$. Calculate the total idle time for the machines in this period.

4 Attempt any two of the following: $10 \times 2 = 20$

(a) A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the storage cost amount to Re. 0.60 per unit per year. The set up cost per run is Rs. 80. Find (i) The optimum run size, (ii) Optimum scheduling period and (ii) The minimum average yearly cost.
(b) A machine owner finds from his past records that the costs per year of maintaining (i.e. operations) a machine whose purchase price is Rs. 6000, are as given below:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (in Rs.)</td>
<td>Operating</td>
<td>1000</td>
<td>1200</td>
<td>1400</td>
<td>1800</td>
<td>2300</td>
<td>2800</td>
<td>3400</td>
</tr>
<tr>
<td>Value (in Rs.)</td>
<td>Resale</td>
<td>3000</td>
<td>1500</td>
<td>750</td>
<td>375</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

Determine at what age, the replacement of the machine is due?

(c) The following failure rates have been observed for a certain type of light bulbs:

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent failing by the end the week</td>
<td>10</td>
<td>25</td>
<td>50</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

There are 1000 bulbs in use and it costs Rs. 10 to replace an individual bulb which has burnt out. If all the bulbs were replaced simultaneously, it would cost Rs. 4 per bulb. It is proposed to replace all bulbs at fixed intervals of time, whether or not they have burnt out and to continue replacing burnt out bulbs as and when they fail. At what intervals, should all the bulbs be replaced? At what group replacement price per bulb would a policy of strictly individual replacement become preferable to the adopted policy?
Attempt any two of the following:

(a) What do you understand by inventory? Give the merits and demerits of inventory? What is inventory control? What are the main objectives of inventory control?

(b) Use dynamic programming to solve the following LPP:

Maximize \( Z = x_1 + 9x_2 \)

Subject to \( 2x_1 + x_2 \leq 25, \quad x_2 \leq 11 \)
and \( x_1, x_2 \geq 0 \)

(c) State Bellman's principle of optimality in reference to dynamic programming problem. Give the basic characteristics of dynamic programming problem.