B. Tech.
(SEM. II) EXAMINATION, 2006-07
MATHEMATICS - II
(SPECIAL CARRYOVER EXAMINATION)

Time : 3 Hours] [Total Marks : 100

Notes : (i) Attempt all questions.
(ii) All questions carry equal marks.

1. Attempt any four parts of the following:
   (a) Solve the first order differential equation

   \[ \frac{dy}{dx} + \frac{y}{\left(1 - x^2\right)^{3/2}} = \frac{x + \sqrt{1 - x^2}}{\left(1 - x^2\right)^2}. \]

   (b) Integrate

   \[ \left(1 + x^2\right) \frac{dy}{dx} + 2xy - 4x^2 = 0. \]

   Obtain equation of the curve satisfying this equation and passing through the origin.

   (c) Find the complete solution of the differential equation

   \[ \left(D^2 - 1\right)y = xe^x + \cos^2 x. \]

   (d) Solve the following simultaneous differential equation

   \[ \frac{dx}{dt} = -uy, \quad \frac{dy}{dt} = wx. \]

   Also show that the point \((xy)\) lies on a circle.

Z-9929] 1 [Contd...
(e) If \( \frac{d^2x}{dt^2} + \frac{g}{b}(x-a) = 0 \) (\( a, b \) and \( g \) being positive constants) and \( x = a' \) and \( \frac{dx}{dt} = 0 \) when \( t = 0 \). Show that

\[
x = a + (a' - a) \cos \left( \frac{g}{\sqrt{b}} t \right).
\]

(f) A particle begins to move from a distance 'a' towards a fixed centre, which repels it with retardation \((\mu x)\). If its initial velocity is \( a\sqrt{\mu} \), show that it will continually approach the fixed centre, but will never reach it.

2 Attempt any two parts of the following:

(a) Prove the orthogonal property of Legendre polynomial

\[
\int_{-1}^{1} P_m(x) P_n(x) dx = \frac{2}{(2n + 1)} \delta_{mn}
\]

where Kronecker delta \( \delta_{mn} \) is

\[
\delta_{mn} = \begin{cases} 
0 & \text{if } m \neq n \\
1 & \text{if } m = n 
\end{cases}
\]

(b) Find the series solution of Bessel's differential equation \( x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0 \)

(c) Prove the recurrent relations:

(i) \( \frac{d}{dx} [x^{-n} J_n(x)] = -x J_{n+1}(x) \)

(ii) \( \frac{2n}{x} J_n(x) = J_{n-1}(x) + J_{n+1}(x) \)

Z-9929] 2 [Contd...
Attempt any four parts of the following:

(a) Find the inverse Laplace transforms of following functions:

\[
\begin{align*}
(i) & \quad \frac{14s + 10}{49s^2 + 28s + 13} \\
(ii) & \quad \frac{1}{s^4 + 4}
\end{align*}
\]

(b) Find the Laplace transform of "Saw-tooth wave" function \( f(t) \) which is periodic with period 1 and defined as \( f(t) = kt \) in \( 0 < t < 1 \).

(c) Find the Laplace transform periodic function

\[
f(t) = \begin{cases} 
   t - t + 2a & 0 < t < a \\
   a < t < 2a &
\end{cases}
\]

(d) Using convolution theorem, find the inverse of the function \( \frac{1}{(s^2 + a^2)^2} \).

(e) Using Laplace transform, evaluate the following integrals

\[
\begin{align*}
(i) & \quad \int_0^\infty \frac{e^{-t} \sin \sqrt{5}t}{t} \, dt \\
(ii) & \quad \int_0^\infty \left( \frac{e^{-2t} - e^{-4t}}{t} \right) \, dt
\end{align*}
\]

(f) Using Laplace transform, solve the equation

\[
L \frac{dI}{dt} + RI = E e^{-at}, \ I(0) = 0
\]

where \( L, R, E \) and \( a \) are constants.

[Contd...]
4 Attempt any two parts of following:
(a) Obtain the Fourier series expansion of
\[ f(x) = \left( \frac{\pi - x}{2} \right) \quad \text{for} \quad (0 < x < 2). \]
(b) If \( f(x) = \sin \left( \frac{\pi x}{L} \right) \) in \( (0 < x < L) \).
Find the Fourier cosine series. Graph the corresponding periodic continuation of \( f(x) \).
(c) Solve the partial differential equation by method of separation of variables
\[ \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0. \]

5 Attempt any two parts of following:
(a) Find the temperature in a bar of length 2 whose each are kept at zero and lateral surface insulated of the initial temperature is
\[ \left[ \sin \left( \frac{\pi x}{2} \right) + 3 \sin \left( \frac{5\pi x}{2} \right) \right]. \]
(b) Find the steady state temperature distribution in a rectangular them plate with its two surfaces insulated and with the condution.
\[ u(0, y) = 0, u(x, 0) = 0, u(a, y) = g(y) \]
\[ u(x, b) = f(x) \]
(c) Solve \[ \frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} \]
assuming that the initial voltage is \[ V_0 \sin \left( \frac{\pi x}{l} \right) \quad V_1(x_0) = 0 \] and \( V = 0 \) at the ends, \( x = 0 \) and \( x = L \) for all \( t \).